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Supergravity, D-brane probes and thermal super Yang-Mills: a comparison

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Abstract: A D3-brane probe in the context of AdS/CFT correspondence at finite temperature is considered. The supergravity predictions for the physical effective couplings of the world-volume gauge theory of the probe brane are compared to those calculated in one-loop perturbation theory in the thermal gauge theory. It is argued that when the Higgs expectation value is much larger than the temperature, the supergravity result must agree with perturbative thermal Yang-Mills. This provides a perturbative test of the Maldacena conjecture. Predictions for the running electric and magnetic effective couplings, beyond perturbation theory are also obtained. Phenomenological applications for universe-branes are discussed. In particular mechanisms are suggested for reducing the induced cosmological constant and naturally obtaining a varying speed of light and a consequent inflation on the universe brane.

Keywords: D-branes, Thermal Field Theory, 1/N Expansion, Cosmology of Theories beyond the SM.

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Contents

1. Introduction

The central elements of CFT/anti-de Sitter (AdS) correspondence [1, 2, 3] are the black D-brane solutions of type II supergravity [4] and their near-horizon geometry [5] along with their microscopic interpretation [6]. In particular, the $(3+1)$-dimensional world-volume of N coinciding, extremal D3-branes is the arena of $\mathcal{N}=4$ supersymmetric SU(N) Yang-Mills (SYM) theory which in the large-N limit, according to the Maldacena conjecture [1], is dual to type IIB superstrings propagating on the near-horizon $AdS_5 \times S^5$ background geometry. There is a further proposal [7] linking the thermodynamics of large-N, $\mathcal{N}=4$ supersymmetric SU(N) Yang-Mills theory with the thermodynamics of Schwarzschild black holes embedded in the AdS space [8]. The classical geometry of black holes with Hawking temperature T encodes the magnetic confinement, mass gap and other qualitative features of large-N gauge theory heated up to the same temperature. At the computational level, the quantity that has been discussed to the largest extent [9, 7, 10]–[13] is the Bekenstein-Hawking entropy which, in the Maldacena limit, should be related to the entropy of Yang-Mills gas at $N \to \infty$ and large 't Hooft coupling $g^2_{\mathrm{YM}}N$.

Turning-on Higgs expectgation values in the gauge theory corresponds in the brane picture to moving around the D3-branes. A useful configuration is one in which one (or a few) D-branes are put a distance r away from the stack of N D-branes (while being kept parallel). In gauge theory this amounts to turning on a Higgs expectation value r/α' breaking the gauge symmetry $\mathrm{SU}(N+1) \to \mathrm{SU}(N) \times \mathrm{U}(1)$.

In the limit of $N \to \infty$ and large 't Hooft coupling λ, the D-brane stack is well described by supergravity with a specific background field configuration. Thus, the world-volume action of the probe brane can be evaluated from the knowledge of the coupling of the world-volume gauge theory to the bulk supergravity fields (see for example [14]). On the other hand at small λ the effective action of the probe brane can be computed in Yang-Mills perturbation theory. At extremality (zero temperature in the gauge theory) $\mathcal{N} = 4$ supersymmetry is known to prohibit renormalization of the two-derivative effective action and this is also visible in the supergravity prediction. There are renormalizations to the four-derivative effective action and the leading supergravity term is expected to be given by the one-loop gauge theory result [15, 16].

Supersymmetry can be broken softly by putting the gauge theory in a heat bath with temperature T. This is expected to be described in supergravity by the near horizon limit of a black D-brane, namely an AdS black-hole whose Hawking temperature is T [7]. Moreover, a probe brane sitting outside of the horizon at a distance r is described by the U(1) part of thermal $\mathcal{N} = 4$ SYM $\mathrm{SU}(N+1) \to \mathrm{SU}(N) \times \mathrm{U}(1)$. Several phenomena as well as the fate of the original scale duality can be still studied in this context [17]–[21].

Here we will study the effective gauge theory action on the probe D3-brane at finite temperature. At strong 't Hooft coupling λ, it will be obtained from supergravity. The relevant data are the classical black-brane solution as well as the world-volume D-brane action. The relevant scales appearing in the theory are the Higgs expectation value u and the temperature T (or rather the thermal wavelength $\sqrt{\lambda}\ T$. From the supergravity point of view, $\pi\sqrt{\lambda}\ T \leq u$. At $\pi\sqrt{\lambda}\ T = u$ corresponds the black-brane horizon. Thus, from supergravity we can calculate the effective potential, the kinetic terms as well as higher derivative terms (for example F^4 terms). This effective action is supposed to be valid as $N \to \infty$ and λ large.

In perturbation theory the relevant diagrams are open string diagrams with a number of boundaries attached to the stack of N D3-branes and a single boundary stack on the probe brane [22]. Their $\alpha' \to 0$ limit provides the appropriate gauge theory diagrams. They integrate out the $\mathrm{SU}(N)$ and massive degrees of freedom.

We argue here, that in the supersymmetry restoration limit $\sqrt{\lambda}\ T \ll u$ the supergravity result has a natural expansion in powers of λ, and can be compared with perturbative Yang-Mills calculations. A similar situation occurs in near extremal calculations of black-hole gray-body factors [23]. The key ingredient for such behavior is supersymmetric non-renormalization theorems valid in the limit where supersymmetry is restored [24].

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Supergravity predicts that the leading contribution to the effective potential comes from three-loops and is proportional to T^8/u^4. As a first test we do the one-loop computation of the vacuum energy by integrating-out the massive open strings, and we finally take the $\alpha' \to 0$ limit. In the relevant supersymmetric limit $\sqrt{\lambda}\, T \ll u$ the one-loop contribution is exponentially suppressed as $e^{-u/T}$. This contribution can be thought-of a non-perturbative contribution due to a solitonic string (see also [25]). For this to be true we must have the following hierarchy: $\sqrt{\lambda} \ll T/u \ll 1$. Thus, up to exponentially suppressed contributions the one-loop calculation gives zero for the potential in the limit $T/u \ll 1$ in accordance with supergravity. The exponentially suppressed contributions will be absent if we treat the fundamental multiplet with mass $\sim u$ (corresponding to the open string stretched between black and probe brane) to be at zero temperature. We can thus formulate the following

Conjecture. *Consider* $\mathcal{N} = 4$ *superYM* $\mathrm{SU}(N+M) \to \mathrm{SU}(N) \times \mathrm{SU}(M) \times \mathrm{U}(1)$ *by a Higgs expectation value* u*, 't Hooft coupling* λ *and* $N \gg M$*. Consider also that the* $\mathrm{SU}(N)$ *part is in a heat bath with temperature* T*. For* $\lambda\, T \ll u$ *the* $\mathcal{O}(N)$ *part of the vacuum energy has a leading behavior* $\sim \lambda^2 T^8/u^4$ *and is given by supergravity. The* $\mathcal{O}(N)$ *part of the vacuum energy can be considered as the vacuum energy induced on the probe brane due to quantum effects on the other brane or the bulk.*

A similar investigation of the kinetic terms for scalars and gauge bosons gives a supergravity derived result which starts at two loops in the limit $T/u \ll 1$. A one-loop calculation gives an exponentially suppressed result (or zero for non-thermal fundamental) , in accordance with the supergravity calculation. Finally, supergravity predicts a one-loop (and higher) contribution to the F^4 couplings of the probe brane. The one-loop open-string/gauge theory calculation is performed and gives agreement with supergravity as expected.

There are two phenomena observed here that may be potentially important for phenomenological purposes.

- **Suppression of vacuum energy on a brane**. It is popular lately to consider our four dimensional universe as a three-brane, embedded in ten dimensional, (partially) compactified spacetime. Moreover, other three-branes may be providing mirror universes. The prototype of this is the Hořava-Witten interpretation of the Heterotic String [26]. One of the important effects in this context is supersymmetry breaking in a mirror brane and its communication in our universe. Here we have a toy model of this situation. In the presence of supersymmetry, the spectator brane is the black-brane while our universe is represented by the probe brane. Both three-branes carry $\mathcal{N} = 4$ supersymmetry. There is only one dynamical scale in the problem: the distance or Higgs expectation value, $M_{\mathrm{DYN}} \sim u$. Spontaneous supersymmetry breaking on the spectator branes is modeled by considering a thermal state (with temperature T). The supersymmetry breaking scale is $M_{\rm SUSY} \sim T$. Standard supertrace formulae imply that when $\mathcal{N} = 4$ supersymmetry is broken the vacuum energy scales as $M_{\rm SUSY}^4$. Here we find that the cosmological constant induced in our universe due to supersymmetry breaking on the spectator brane is much smaller: it scales as $\Lambda \sim M_{\rm SUSY}^8/M_{\rm DYN}^4$.[1] Moreover, it is expected that the extra contributions to the vacuum energy on the universe brane due to loops of brane fields will have extra suppression factors of the coupling constant. If instead we use a stack of black-branes whose extremal limit are branes at an orbifold singularity with $1 \leq \mathcal{N} \leq 4$ then there is a similar behavior in the vaccum energy. There is also a hint that this supression of the vacuum energy would persist for the case of $\mathcal{N} = 2$ world-volume supersymmetry on the probe.

- **Induction of field-dependent or time-varying speed of light on a brane**. When a probe brane moves in the gravitational field of another black brane, the induced field theory on the probe brane although Lorentz invariant, has a field dependent velocity of light. In the gauge theory picture this effective velocity of light is due to thermal quantum effects. In the simple example we analyze here this velocity of light dependents on the distance (which is also a dynamical scalar field of the probe brane) to the black-brane. In the case discussed in this paper, the probe brane is outside the horizon of the black brane. There may be situations where the effective velocity of light is time dependent and that can be achieved by taking the brane inside the horizon and performing the appropriate analytic continuation. A time-varying speed of light can be an alternative to inflation and can thus provide different way to solve the flatness problem in cosmology, [29].

The structure of this paper is as follows; In section 2 we describe the basic black-brane solutions we will be using here as well as we evaluate the wold-volume probe action in such backgrounds. In section 3 we focus on three-branes, we take the near-horizon limit and gravitationally derive the thermal Yang-Mills potential. In section 4, we perform the one-loop computation of the potential in open string theory and by taking the α'-limit in thermal gauge-theory. This agrees with the gravitational calculation up to exponentially suppressed terms. In section 5 we do a similar analysis for two and four derivative effective couplings on the probe brane. Finally in section 2 we discuss potential phenomenological applications of the phenomena discussed here. In the appendix a careful evaluation of the RR gauge field in black-brane configurations is given.

[1]For $M_{\rm SUSY} \sim 10\,{\rm TeV}$ and $M_{\rm DYN} \sim M_{\rm Planck}$ we obtain $\Lambda \sim 10^{-120} M_{\rm Planck}^4$.

2. Black Dp-branes

We consider now the background geometry (in the string frame) of a near-extremal black hole describing a number of coinciding Dp-branes [4]:

$$ds_{10}^2 = \frac{-f(r)dt^2 + d\vec{x}\cdot d\vec{x}}{\sqrt{H_p(r)}} + \sqrt{H_p(r)}\left(\frac{dr^2}{f(r)} + r^2 d\Omega_{8-p}^2\right), \tag{2.1}$$

where

$$H_p(r) = 1 + \frac{L^{7-p}}{r^{7-p}}, \qquad f(r) = 1 - \frac{r_0^{7-p}}{r^{7-p}}. \tag{2.2}$$

The parameters L and r_0 determine the AdS throat size and the position of horizon, respectively. They are related to the ADM mass M and the (integer) Ramond-Ramond charge N in the following way:

$$M = \frac{\Omega_{8-p}V_p}{2\kappa_{10}^2}\left[(8-p)r_0^{7-p} + (7-p)L^{7-p}\right], \tag{2.3}$$

$$N = \frac{(7-p)\Omega_{8-p}}{2\kappa_{10}^2 T_p} L^{(7-p)/2}\sqrt{r_0^{7-p} + L^{7-p}}, \tag{2.4}$$

where Ω_n is the volume of a unit n-dimensional sphere S^n,

$$\Omega_n = \frac{2\pi^{(n+1)/2}}{\Gamma((n+1)/2)}, \tag{2.5}$$

and V_p is the common p-dimensional D-brane (flat) volume. The relations (2.3) and (2.4) involve the D-brane tension T_p and the 10-dimensional gravitational constant κ_{10} which are determined by the string coupling g_s and the string tension α' as follows:

$$T_p = \frac{1}{(2\pi)^p \alpha'^{(p+1)/2} g_s}, \qquad 2\kappa_{10}^2 = (2\pi)^7 \alpha'^4 g_s^2. \tag{2.6}$$

The RR charge N is quantized, with each D-brane carrying a unit charge so that N is equal to the number of D-branes. Note that in the extremal case ($r_0 = 0$), $M = NV_pT_p$. Finally,

$$L^{7-p} = \sqrt{\left(\frac{2\kappa_{10}^2 T_p N}{(7-p)\Omega_{8-p}}\right)^2 + \frac{1}{4}r_0^{2(7-p)}} - \frac{1}{2}r_0^{7-p}. \tag{2.7}$$

The RR charge is the source of the p-form field

$$C_{012\cdots p}(r) = \frac{2\kappa_{10}^2 T_p N}{\Omega_{8-p}(7-p)(r^{7-p} + L^{7-p})} = \sqrt{1 + \frac{r_0^{7-p}}{L^{7-p}}}\,\frac{H_p(r) - 1}{H_p(r)}. \tag{2.8}$$

All other components vanish, except in the case of $p = 3$, when the self-duality condition

$$F_{\mu_1\cdots\mu_5} = \frac{1}{5!\sqrt{\det g}}\epsilon_{\mu_1\cdots\mu_5\nu_1\cdots\nu_5}F^{\nu_1\cdots\nu_5} \tag{2.9}$$

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requires non-zero p-form components in the transverse directions. Since there are discrepancies in the literature, we discuss the p-form solutions in more detail in the appendix. There is also a dilaton background (constant for $p = 3$):

$$e^{\phi} = H_p^{(3-p)/4}(r)\,. \tag{2.10}$$

By using standard methods of black hole thermodynamics, it is straightforward to determine the Hawking temperature, chemical potential and entropy corresponding to the solution (2.1), (2.8) and (2.10). They are respectively:

$$\begin{aligned} T &= \frac{7-p}{4\pi}\frac{r_0^{(5-p)/2}}{\sqrt{r_0^{7-p}+L^{7-p}}}\,, \\ \Phi &= V_p T_p\,\frac{L^{(7-p)/2}}{\sqrt{r_0^{7-p}+L^{7-p}}}\,, \\ S &= \frac{4\pi\Omega_{8-p}V_p}{2\kappa_{10}^2}\,r_0^{(9-p)/2}\sqrt{r_0^{7-p}+L^{7-p}}\,, \end{aligned} \tag{2.11}$$

We consider now a Dp-brane probing the above solution, with zero background values for all other fields. In this case, the D-brane probe action is[2]

$$\Gamma_p = T_p e^{-\phi}\int\sqrt{\det\hat{g}} + T_p\int\hat{C}\,, \tag{2.12}$$

where we have also set the world-volume gauge field strength to zero, $F_{\alpha\beta} = 0$. Using the metric (2.1), the p-form (2.8) and the dilaton (2.10), we obtain the static potential [22]

$$V(r) = V_pT_p\left[\frac{\sqrt{f(r)}}{H_p(r)} + C(r)\right] = V_pT_p\left[\frac{\sqrt{f(r)}}{H_p(r)} + \sqrt{1+\frac{r_0^{7-p}}{L^{7-p}}}\,\frac{H_p(r)-1}{H_p(r)}\right], \tag{2.13}$$

where $C(r) \equiv C_{012\cdots p}(r)$. The values of the potential at infinity and at the horizon are, respectively,

$$V(\infty) = V_pT_p\,, \qquad V(r_0) = \Phi\,, \tag{2.14}$$

We can expand the interaction potential $V^{\rm int}(r) = V(r) - V(\infty)$ at large r, to obtain

$$\begin{aligned} \frac{V^{\rm int}(r)}{V_pT_p} &= \left[L^{7-p}\left(\sqrt{1+\frac{r_0^{7-p}}{L^{7-p}}}-1\right) - \frac{1}{2}r_0^{7-p}\right]\frac{1}{r^{7-p}} - \\ &\quad -\frac{1}{8}\left[8L^{2(7-p)}\left(\sqrt{1+\frac{r_0^{7-p}}{L^{7-p}}}-1-\frac{1}{2}\frac{r_0^{7-p}}{L^{7-p}}\right)r_0^{2(7-p)}\right]\frac{1}{r^{2(7-p)}} + \mathcal{O}(r^{-3(7-p)}) \end{aligned} \tag{2.15}$$

[2] There are also curvature depended couplings. These give zero contribution for the background at hand.

The leading long-distance term can be understood as follows: it is due to the classical interaction of the extremal probe with the non-extremal collection of p-branes. This interaction is proportional [14] to $Q_{\rm probe}(M_p - NV_pT_p)$. An important point here is that the mass M_p, felt by the Dp-brane is not the same as the thermodynamic mass (2.3). From (2.15) we obtain

$$\frac{M_p}{V_p} = \frac{(7-p)\Omega_{8-p}}{2\kappa_{10}^2}\left[L^{7-p} + \frac{1}{2}r_0^{7-p}\right] \tag{2.16}$$

and we have also $Q_{\rm probe} = V_pT_p$. Thus,

$$V^{\rm int} = -\frac{2\kappa_{10}^2}{(7-p)\Omega_{8-p}}\frac{Q_{\rm probe}(M_p/V_p - NT_p)}{r^{7-p}} + \cdots \tag{2.17}$$

Different D-brane probes feel different masses. The interaction potential for a D$(p-2n)$-brane probe is

$$\begin{aligned} V_{p,n}^{int}(r) &= V_{p-2n}T_{p-2n}\left[\sqrt{f(r)}H_p(r)^{-1+n/2} - 1\right] = \\ &= -V_{p-2n}T_{p-2n}\frac{\left(1-\frac{n}{2}\right)L^{7-p} + \frac{1}{2}r_0^{7-p}}{r^{7-p}} + \mathcal{O}(r^{-2(7-p)}) \end{aligned} \tag{2.18}$$

When $n \neq 0$ there is no interaction due to the exchange of a RR field and the large distance interaction is

$$V_{p,n}^{\rm int}(r) = -\frac{2\kappa_{10}^2}{(7-p)\Omega_{8-p}}\frac{Q_{\rm probe}(M_{p,n}/V_p)}{r^{7-p}} + \cdots \tag{2.19}$$

with the mass seen by the D$(p-2n)$-branes

$$\frac{M_{p,n}}{V_p} = \frac{(7-p)\Omega_{8-p}}{2\kappa_{10}^2}\left[\left(1-\frac{n}{2}\right)L^{7-p} + \frac{1}{2}r_0^{7-p}\right] \tag{2.20}$$

Note that for a $(p-4)$-brane probe, $n=2$, the apparent mass vanishes at extremality, as expected, since the system of p- and $(p-4)$-branes does not break all of supersymmetry. Also, note that for $n>2$, the interaction can become repulsive near extremality since for $r_0 \ll L$ we obtain that the leading large distance interaction is

$$V_{p,n=0}^{\rm int}(r) = -V_pT_p\frac{r_0^{2(7-p)}}{8L^{7-p}}\frac{1}{r^{7-p}}\,, \qquad V_{p,n}^{\rm int}(r) = \left(\frac{n}{2}-1\right)\frac{L^{7-p}V_{p-2n}T_{p-2n}}{r^{7-p}} \tag{2.21}$$

plus terms that are suppressed by extra powers of $\frac{L^{7-p}}{r^{7-p}}$, $\frac{r_0^{7-p}}{L^{7-p}}$. Finally, the potential for a fundamental string probe is similar to that of a $(p-2)$-brane.

3. D3-branes, the near-horizon limit and the thermal Yang-Mills potential

The case of D3-branes is particularly interesting because the world-volume action of N coinciding D-branes involves a four-dimensional $\mathcal{N} = 4$ supersymmetric SU(N) Yang-Mills theory. Moreover, in this case there is a natural correspondence (at all scales) with supergravity: according to the Maldacena conjecture [1], the large-N limit of this gauge theory is related to the near-horizon AdS geometry of the extremal ($r_0 = 0$) black D3-brane solution (2.1). Witten [7] has exploited the AdS/SYM correspondence in order to study the large-N dynamics of non-supersymmetric SYM, with $\mathcal{N} = 4$ supersymmetries broken by non-zero temperature effects. According to this proposal, the non-extremal solution (2.1) may be used to study SYM at T identified with the Hawking temperature (2.11) as long as $T \ll 1/L$, so that the metric remains near-extremal ($r_0 \ll L$). In the near-horizon limit, $\alpha' \equiv l_s^2 \to 0$ at $u \equiv r/\alpha'$ and T fixed, the solution (2.1) describes an AdS-Schwarzschild black hole [8]:

$$ds^2 = l_s^2 \left[\frac{u^2}{R^2}(-f(u)dt^2 + d\vec{x}\cdot d\vec{x}) + R^2 \frac{du^2}{u^2 f(u)} + R^2 d\Omega_5^2 \right] + \mathcal{O}(l_s^4)\,, \tag{3.1}$$

where

$$f(u) = 1 - \frac{u_0^4}{u^4}\,, \qquad R^4 \equiv 4\pi g_s N = \lambda\,, \qquad u_0 = \pi T R^2\,, \tag{3.2}$$

where λ is the 't Hooft coupling. The limiting value of the four-form (2.8) is

$$C_{0123} = 1 + l_s^4 \left(\frac{(\pi T R)^4}{2} - \frac{u^4}{R^4} \right) + \mathcal{O}(l_s^8)\,, \tag{3.3}$$

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A D3-brane probe in the bulk of the AdS space corresponding to N background D3-branes can be thought of as a realization of SU($N+1$) gauge theory in the SU(N)$\times$U(1) symmetric Higgs phase. In the following, we will examine the potential induced on the probe brane in the near-horizon limit and we will eventually compare it with the gauge theory calculation.

To that end, we will use the following expansions in the string length scale l_s:

$$\begin{aligned} L^4 &= R^4 l_s^4 \left(1 - \frac{1}{2}\pi^4 R^4 T^4 l_s^4\right) + \mathcal{O}(l_s^{12})\,, \\ r_0 &= \pi T R^2 l_s^2 \left(1 + \frac{1}{4}\pi^4 T^4 R^4 l_s^4 + \mathcal{O}(l_s^8)\right)\,. \end{aligned} \tag{3.4}$$

which follow from relations written in the previous section.

Taking the limit in the static potential (2.13), we obtain

$$V(u) = V_3 T_3 \left\{ 1 + l_s^4 \frac{u^4}{R^4} \left[\sqrt{1 - \left(\frac{\pi T R^2}{u}\right)^4} - 1 + \frac{1}{2}\left(\frac{\pi T R^2}{u}\right)^4 \right] + \mathcal{O}(l_s^8) \right\} \tag{3.5}$$

so that the interaction energy is

$$V^{\rm int}(u) = V(u) - V_3 T_3 = \frac{V_3}{(2\pi)^3 g_s}\frac{u^4}{R^4}\left[\sqrt{1-\left(\frac{\pi T R^2}{u}\right)^4} - 1 + \frac{1}{2}\left(\frac{\pi T R^2}{u}\right)^4\right] + \mathcal{O}(l_s^4) \tag{3.6}$$

and has a smooth limit as $l_s \to 0$. Since the probe is BPS, $V^{\rm int}(\infty) = 0$ [21].

The potential in (3.6) according to the Maldacena conjecture has a direct interpretation in the context of SU$(N+1)$ $N=4$ gauge theory at finite temperature. We consider SU$(N+1)$ $N=4$ gauge theory and a Higgs expectation value u that breaks the gauge symmetry to SU(N) $\times$ U(1). At finite temperature, supersymmetry is broken. We consider now the quantum effective action for the U(1) factor, obtained by integrating out all SU(N) as well as massive degrees of freedom. This has an expansion in powers of $1/N$. The leading piece is $\mathcal{O}(N)$. At large 't Hooft coupling λ this should be given by supergravity as in (3.6). We will rewrite it in terms of gauge theory variables as

$$V^{\rm int}(u) = \frac{N V_3}{2\pi^2}\frac{u^4}{\lambda^2}\left[\sqrt{1-\frac{\pi^4\lambda^2 T^4}{u^4}} - 1 + \frac{1}{2}\frac{\pi^4\lambda^2 T^4}{u^4}\right] + \mathcal{O}(\lambda^{-3/2}) \tag{3.7}$$

This form should be compared with gauge theory for large Higgs expectation value $u \gg T$. For u close to the horizon, there is a non-trivial map between the supergravity and the gauge theory variable [22]. This is obtained by matching the form of the world-volume kinetic terms between supergravity and gauge theory. The D-brane coordinate ρ is related to the supergravity coordinate u as

$$\rho^2 = u^2 + \sqrt{u^4 - u_0^4}\,. \tag{3.8}$$

Defining as in [12] the mass scale $M = (\rho - u_0)/R^2$ that controls the approach to the horizon we obtain[3]

$$V^{\rm int} = -\frac{1}{4}\pi^2 N V_3 T^4\left[1 - \frac{4}{\pi}\frac{M}{T} - \frac{10}{\pi^2}\frac{M^2}{T^2} - \frac{20}{\pi^3}\frac{M^3}{T^3} + \mathcal{O}\left(\frac{M^4}{T^4}\right)\right]. \tag{3.9}$$

As mentioned before the potential in (3.7) is supposed to be valid in the limit of large 't Hooft coupling. We can, however, consider the limit in which the Higgs expectation is much larger than the thermal wavelength, $u \gg \sqrt{\lambda}T$. In this limit, supersymmetry is broken very softly. Expanding the supergravity generated potential we obtain

$$V^{\rm int}(u) = \frac{\pi^2}{2} N V_3 T^4 \sum_{m=1}^{\infty}\frac{(2m-1)!!}{2^{m+1}(m+1)!}\left(\frac{\pi\lambda T^2}{u^2}\right)^{2m}. \tag{3.10}$$

[3] This is in agreement with [12]. The potential here differs however in the large u limit. This is because we chose the constant in the four-form (before taking the near-horizon limit) to be such that it vanishes at infinity. A different prescription was used in [12].

We observe that the large u expansion is equivalent here to an expansion in the 't Hooft coupling. The leading term $m = 1$ is a three-loop term. This is suggestive that although the result is strictly speaking valid for strong 't Hooft coupling, it might still be reliable also in perturbation theory.

This situation is reminiscent of an analogous phenomenon in the near-extremal calculations of grey-body factors in D-brane black-holes [23]. There also, a result that is valid at strong 't Hooft coupling agrees with Yang-Mills perturbation theory. The reason was traced at specific non-renormalization theorems due to softly broken supersymmetry, [24]. A similar phenomenon is occurring here. For this to work, contributions that are higher order in α' should be suppressed by extra powers of $\lambda T^2/u^2$. The next order term in the bulk action is of order α'^3. To calculate its contribution to the potential we need the α' corrected form of the four-form away from the near-horizon limit. Such an investigation is under way and will provide further evidence for the above.

The diagrams relevant here, that are leading in N, at higher orders in the 't Hooft coupling have one boundary stuck at the probe brane and the rest are attached to the stack of N D3-branes. Thus, they scale as $N(g_s N)^{B-2} \sim N\lambda^{B-2}$ where B is the total number of boundaries. Moreover, the supergravity result implies that only diagrams with $B = 2m + 2$, $m > 0$ contribute. In the gauge theory this implies that contributing diagrams appear at order $m+1$, starting at three loops. Written in the double line notation of 't Hooft they have $2m+1$ index loops with N circulating colors and one with one color. Thus, they can be represented as a disk with $2m + 1$ holes. Cutting the diagrams appropriately we can see that SU(N) degrees of freedom circulate in intermediate channels. These are responsible for producing the temperature dependent factors and in particular the power-like behaviour of the potential.

In analogy with grey-body factor calculations we can conjecture that the potential (3.7) can be compared with perturbative thermal Yang-Mills calculations. In particular this implies that there are no one-loop and two-loop contributions, and the first non-trivial term is coming in at three-loops. The leading contribution is $\sim \lambda^2\, T^8/u^4$.

In the next section we will do the one-loop computation and show that in the large u limit it is exponentially suppressed. These exponential contributions are "non-perturbative" from our point of view and can be interpreted as due to solitonic loops. Thus, up to such terms, the gauge theory will give a zero contribution at one loop. Moreover, there is independent evidence that the next term comes from three loops and behaves like T^8/u^4. In [30] the theory analyzed is SU(2) $\mathcal{N} = 2$ super YM and the relevant contribution was found from a two-loop diagram with an effective one-loop generated vertex. This amounts to a three-loop contribution in the original SU(2) theory.

There is an alternative way to implement the perturbative calculation: consider a thermal ensemble in the SU(N) part but treat the massive degrees of freedom as though they were not thermal. Then the one loop calculation in the gauge theory will give zero both for the potential and the kinetic terms. This will be shown explicitly in the next section. The difference with the previous calculation is that there will be no exponentially suppressed terms here.

The above can produce a stringent test of the Maldacena conjecture that can be performed in Yang-Mills perturbation theory. One needs to evaluate the effective potential of the U(1) scalar in $\mathrm{SU}(N+1) \to \mathrm{SU}(N) \times \mathrm{U}(1)$ theory at two and three loops. The relevant $\mathcal{O}(N^2)$ two-loop diagram is shown in fig. 1 (we use the notation of fig. 2). This sumarises contributions of scalars, fermions and ghosts. Note that there is no relevant double-bubble diagram. According to our conjecture this should give only exponentially small contributions at large u, (or zero iff the massive fundamental is not thermalized). The diagram includes two massive external propagators as well as a massless internal one. This can be written as a one-loop diagram of the $\mathrm{SU}(N)$ degrees of freedom where instead of the tree propagator we use the one-loop corrected propagator where only massive states go around the loop (fig. 1). However, the results of the next section imply that such corrections to the propagators after summed over the various states are exponentially suppressed. This is in accord with our expectations that the two-loop contribution to the potential is exponentially suppressed.

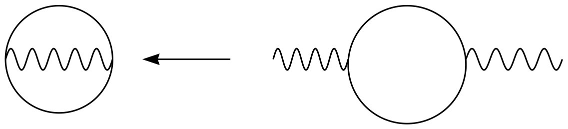

Figure 1: $\mathcal{O}(N^2)$ two-loop contribution to the effective potential.

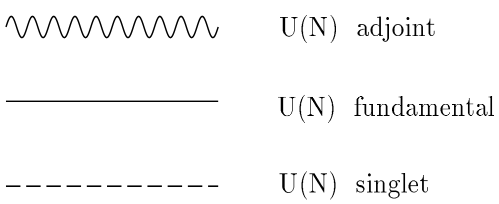

Figure 2: Line notation for the Feynman diagrams.

The appropriate field theory diagrams that contribute to three-loop order are shown in fig. 3. It would be extremely interesting to perform the field theory calculation and reproduce the leading term in (3.10).

The conjecture above is also valid in the case where $\mathrm{SU}(N+M) \to \mathrm{SU}(N) \times \mathrm{SU}(M) \times \mathrm{U}(1)$ with $N \gg M$. The probe action must be replaced by the appropriate non-abelian generalization [31] and the potential is M^2 times the one calculated in the U(1) case.

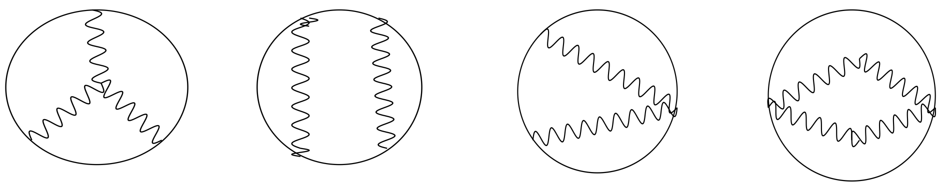

Figure 3: $\mathcal{O}(N^3)$ three-loop contributions to the effective potential.

4. Perturbation theory and the potential

In this section we will study the static potential further and we will try to make contact with gauge theory perturbation theory.

We will first consider an open string theory evaluation of the static potential. This can be done by integrating out in perturbation theory the stretched strings between N D3-branes and a D3-probe at distance r. The one-loop free-energy at finite temperature is [32]

$$F\left(\frac{\beta}{l_s},\frac{r}{l_s}\right) = -\frac{V_3 N}{16\pi^2\alpha'^2}\int_0^\infty \frac{dt}{t^3}e^{-\frac{r^2 t}{2\pi\alpha'}}\left[\vartheta_3\left(\frac{i\beta^2}{4\alpha'\pi t}\right)\frac{1}{2}\frac{\vartheta_3^4(it)-\vartheta_4^4(it)}{\eta^{12}(it)} - \right.$$
$$\left. - \vartheta_4\left(\frac{i\beta^2}{4\alpha'\pi t}\right)\frac{1}{2}\frac{\vartheta_2^4(it)}{\eta^{12}(it)}\right] \tag{4.1}$$

Using supersymmetry and

$$\vartheta_3\left(\frac{i\beta^2}{4\alpha'\pi t}\right) - \vartheta_4\left(\frac{i\beta^2}{4\alpha'\pi t}\right) = 2\vartheta_2\left(\frac{i\beta^2}{\alpha'\pi t}\right) \tag{4.2}$$

we obtain

$$F = -\frac{V_3 N}{16\pi^2\alpha'^2}\int_0^\infty \frac{dt}{t^3}e^{-\frac{r^2 t}{2\pi\alpha'}}\vartheta_2\left(\frac{i\beta^2}{\alpha'\pi t}\right) f_{\text{open-string}}(t) \tag{4.3}$$

with

$$f_{\text{open-string}}(t) = \frac{1}{2}\frac{\vartheta_2^4(it)}{\eta^{12}(it)} = 8 + \mathcal{O}(e^{-\pi t})$$
$$f_{\text{open-string}}\left(\frac{1}{t}\right) = t^{-4} f_{closed-string}(t) = \frac{t^{-4}}{2}\frac{\vartheta_4^4(it)}{\eta^{12}(it)} = \frac{t^{-4}}{2}\left(e^{\pi t} + \mathcal{O}(e^{-\pi t})\right) \tag{4.4}$$

Near $t \to \infty$ the integrand behaves as

$$\frac{1}{\beta}\int^\infty \frac{dt}{t^{5/2}}e^{-\frac{r^2 t}{2\pi^2\alpha'}}, \tag{4.5}$$

which is convergent. If we consider the expansion of the exponential in powers of r^2 then the terms after the first two diverge. Near $t \to 0$ the integrand behaves as

$$\int_0 t dt e^{\frac{\pi}{t}\left(1-\frac{\beta^2}{4\alpha'\pi}\right)} \tag{4.6}$$

and we clearly see the signal of the open string Hagedorn transition at $\beta_H = 2\sqrt{\pi\alpha'}$. We will thus assume from now on that $\beta > \beta_H$.

Among the three scales r, β, α' we can make two dimensionless parameters, $\tilde{\beta} = \beta/\sqrt{\alpha'}$ and $\tilde{r} = r/\sqrt{\alpha'}$. We would like to find the behavior of F as a function of $\tilde{\beta}$ and $\tilde{r}$.

When $\tilde{\beta}\tilde{r} \gg 1$ the integral can be evaluated by saddle point. The position of the saddle point is at $t_0 = \beta/\sqrt{2}r$. For $t_0 > 1$, we use $f_{\text{open-string}}$ while for $t_0 < 1$ we use $f_{\text{closed-string}}$. We obtain:

- $\tilde\beta\tilde r \gg 1$ and $\beta > r$

$$F = \frac{V_3 N}{(2\pi)^{1/4} 2\pi^2} \sqrt{\frac{r^3}{\beta^5 \alpha'^3}} e^{-\frac{\beta r}{\sqrt{2\pi}\,\alpha'}} f_{\rm open-string}\left(i\sqrt{\frac{\pi}{2}\frac{\beta}{r}}\right) ; \qquad (4.7)$$

- $\tilde\beta\tilde r \gg 1$ and $\beta < r$

$$F = \frac{V_3 N}{8(2\pi)^{1/4}} \sqrt{\frac{\beta^3}{r^5 \alpha'^3}} e^{-\frac{\beta r}{\sqrt{2\pi}\,\alpha'}} f_{\rm closed-string}\left(i\sqrt{\frac{2}{\pi}\frac{r}{\beta}}\right) . \qquad (4.8)$$

These are instanton contributions. The instanton is the world-sheet of an open string stretched a distance r and wound around the temporal circle. This gives a configuration with area βr and action $S_{\rm inst} \sim \beta r/\alpha'$. The determinant factor is in fact the determinant on the above-mentioned cylinder with (real) modulus $t \sim \beta/r$ of the open string fluctuations. When $t > 1$ they are best described by open string states while when $t < 1$ they are best described by closed string states.

Since $\beta > \beta_H \to \tilde\beta > 2\sqrt{\pi}$, the only other corner in the $\tilde\beta$,$\tilde r$ plane to investigate is $\tilde\beta\tilde r < 1$. In the small r region the massive modes of the string are subleading. The reason is that as $r \to 0$ the mass gap vanishes for the massless states and it is those that dominate the behavior of the free energy. The dominant contribution is [33, 12]

$$F = V_3(2N)\left[-\frac{\pi^2}{6\beta^4} + \frac{r^2}{4\pi\beta^2\alpha'^2} - \frac{r^3}{3\pi^2\sqrt{2\pi}\beta\alpha'^3} + \cdots\right] , \qquad (4.9)$$

which describes accurately the behavior of the free energy in the region $\tilde\beta \gg 1$, $\tilde\beta\tilde r \ll 1$. This is the region with a temperature much lower than the string scale and a Higgs expectation value much lower than the temperature when both are measured in string units. The behavior is indeed that of field theory. The leading term is the change of the Yang Mills free energy $\pi^2 V_2 N^2 T^4$ as $N \to N+1$ and the subleading term generates a mass for the u-scalar, $u = r/\alpha'$, $m_u \sim g_s T$. Moreover, the potential will force r to relax back to $r = 0$.

We will now go to the near-horizon limit: $u = r/\alpha'$ and β fixed.

$$F_M = -\frac{V_3 N}{16\pi^2\alpha'^2}\int_0^\infty \frac{dt}{t^3} e^{-\frac{u^2\alpha' t}{2\pi}} \vartheta_2\left(\frac{i\beta^2}{\alpha'\pi t}\right) f_{\rm open-string}(t) . \qquad (4.10)$$

After changing the variable $t \to t/\alpha'$,

$$\begin{aligned} F_M &= -\frac{V_3 N}{16\pi^2}\int_0^\infty \frac{dt}{t^3} e^{-\frac{u^2 t}{2\pi}} \vartheta_2\left(\frac{i\beta^2}{\pi t}\right) f_{\rm open-string}\left(\frac{t}{\alpha'}\right) \\ &= -\frac{V_3 N}{\pi^2}\int_0^\infty \frac{dt}{t^3} e^{-\frac{u^2 t}{2\pi}} \vartheta_2\left(\frac{i\beta^2}{\pi t}\right) + \mathcal{O}(\alpha') \\ &= -\frac{V_3 N}{\beta^4}\int_0^\infty \frac{dt}{t^3} e^{-\frac{u^2\beta^2 t}{2\pi^2}} \vartheta_2\left(\frac{i}{t}\right) + \mathcal{O}(\alpha') . \end{aligned} \qquad (4.11)$$

Note that the Hagedorn boundary has been pushed to $T \to \infty$. There are essentially two distinct regions:

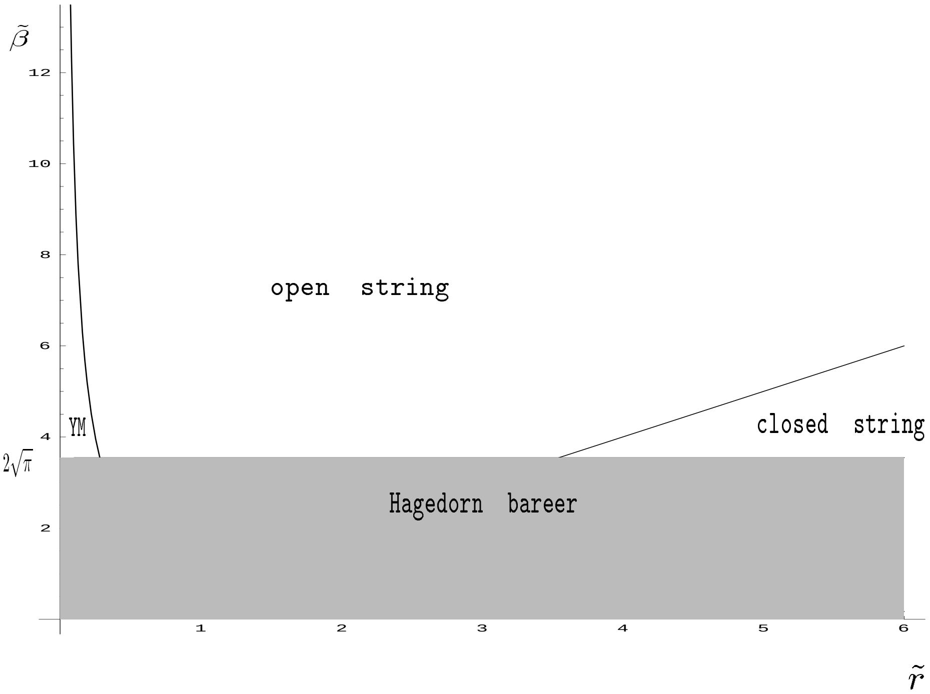

Figure 4: The various regions of the one-loop effective potential.

- $\beta u \gg 1$. It corresponds to the open solitonic string region with

$$F = \frac{8V_3 N}{(2\pi)^{1/4}\pi^2}\sqrt{\frac{u^3}{\beta^5}}e^{-\frac{\beta u}{\sqrt{2\pi}}} \tag{4.12}$$

Here the Higgs expectation value is much larger than the temperature. Thus, integrating out these massive modes gives exponentially suppressed contributions. The supersymmetry breaking parameter, namely the temperature, is much smaller than the scale u of the $\mathcal{N} = 4$ supersymmetric theory. Thus, the supersymmetric non-renormalization theorem for the potential is true to exponential accuracy.

- $\beta u \ll 1$.

$$\begin{aligned} F = & -\frac{V_3(2N)\pi^2}{6\beta^4}\left[1 - \frac{3u^2\beta^2}{2\pi^3} + \frac{u^3\beta^3}{\sqrt{2\pi}\pi^4} - \frac{15\log 2}{4}\frac{u^4\beta^4}{\pi^6} - \right. \\ & \left. - \frac{u^6\beta^6}{\pi^9}\sum_{n=0}^{\infty}\left(\frac{u^2\beta^2}{2\pi^2}\right)^n \binom{\frac{3}{2}+n+2}{n+3}(1-2^{-(2n+2)})\zeta(2n+3)\right] \\ = & V_3(2N)\left[-\frac{\pi^2}{6\beta^4} + \frac{u^2}{4\pi\beta^2} - \frac{u^3}{3\pi^2\sqrt{2\pi}\beta}\cdots\right]. \end{aligned} \tag{4.13}$$

Here the Higgs expectation value is much smaller than the temperature. The theory is close to unbroken Yang Mills as indicated by the free energy. Supersymmetry is completely broken and this is reflected in the form of the free energy. The minimum of the potential is again at $u = 0$. This is true for any temperature since the derivative of the potential with respect to u is proportional to u times an integral of a non-negative integrand. The only other extremum is at $u = \infty$ but this is a maximum. Thus, the potential is monotonically decreasing from $u = \infty$ to $u = 0$.

The form of the potential for large temperature (4.13) agrees qualitatively with the one obtained by supergravity. In this region we do not expect quantitative agreement, but it is obvious that the one-loop contributions has similar features to the strong coupling (supergravity) result. In this limit the vacuum energy has a leading piece which is the supersymmetry breaking scale (temperature) to the fourth power, as expected in a non-supersymmetric theory.

The situation is different at large distances, or large Higgs expectation values. The one-loop results is exponentially suppressed, and one reason for this is approximate supersymmetry in this limit. This can be clearly seen by our computation. Separately bosons and fermions produce a temperature-independent polynomial piece scaling as r^4 at large r plus exponentially small contributions. The polynomial piece cancels between fermions and bosons and we are left with the exponentially small contribution. In general this is an expected feature of softly or spontaneously broken supersymmetry. A different way of viewing this behavior is to state that the object (open string) going around the loop has a finite size and thus gives an exponentially small contribution. From the point of view of supergravity it behaves as a soliton, and it is not visible in the long distance expansion.

It is obvious from our calculation that if the massive open string (or in the $\alpha' \to 0$ limit the massive fundamental multiplet) is at zero temperature then the contribution to the potential will be zero.

The potential (vacuum energy) is generically expected to scale as the larger mass in the fourth power, namely r^4. From supergravity we obtain the prediction that the leading contribution is T^8/r^4 which is strongly suppressed.

We conclude that the supergravity calculation provides us with the physical (non-Wilsonian) effective action of the U(1) gauge theory after integrating out the SU(N) plus the massive degrees of freedom. Moreover, exponentially small contributions in perturbation theory are not visible in the supergravity result. This suggests that the proper context is to take only the SU(N) part to be in a thermal bath. The probe brane is certainly at zero temperature and our analysis indicates that at large distance the ultra-massive string can also be taken to be at zero temperature.

The quantum contributions of the world-volume fields have not been taken into account. In the large-N limit they turn out to be subleading.

5. Kinetic and quartic terms

We will now expand the D3-brane action keeping also (bosonic) quadratic terms in derivatives. We obtain

$$S_{D3} = T_3 \int \sqrt{-\det(\hat{g} + (2\pi\alpha')F - B)} + T_3 \int \hat{C} = S_0 + S_1^s + S_1^F + \mathcal{O}(v^4)\,, \tag{5.1}$$

with S_0 being the potential that we already discussed. Here S_1^s represents the kinetic terms for the scalars,

$$S_1^s = \frac{T_3}{2} \int d^4x \left[\frac{1}{f(r)} \partial r \cdot \partial r + r^2 h_{\alpha\beta} \partial y^\alpha \cdot \partial y^\beta \right]. \tag{5.2}$$

where

$$\partial\phi_1 \cdot \partial\phi_2 = \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \equiv -\frac{1}{\sqrt{f(r)}} \partial_0 \phi_1 \cdot \partial_0 \phi_2 + \sqrt{f(r)} \sum_{i=1}^{3} \partial_i \phi_1 \cdot \partial_i \phi_2 \tag{5.3}$$

and S_1^F are the gauge kinetic terms,

$$\begin{aligned} S_F^1 &= (\pi\alpha')^2 T_3 \int d^3x \frac{1}{\sqrt{f(r)}} \left[\tilde{g}^{\mu\rho} \tilde{g}^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} \right] \\ &= 2(\pi\alpha')^2 T_3 \int d^3x \left[\frac{1}{\sqrt{f(r)}} \vec{E}^2 + \sqrt{f(r)} \vec{B}^2 \right], \end{aligned} \tag{5.4}$$

where as usual $E_i = F_{0i}$, $B_i = \epsilon_{ijk} F_{jk}/2$. We have also set $B_{\mu\nu} = 0$. Note that the velocity of light on the 3-brane is r-dependent. From the formulae above we can ascertain that

$$c_{\text{eff}} = \sqrt{f(r)} = \sqrt{1 - \left(\frac{r_0}{r}\right)^4} \tag{5.5}$$

For a brane next to the horizon, any time dependent fluctuation freezes, since the velocity of light vanishes there.

To see this, set $\epsilon = r - r_0 \to 0$ and

$$\begin{aligned} L_{kin}(r) \sim -\frac{\dot{r}^2}{f^{3/2}} + \frac{(\nabla r)^2}{f^{1/2}} \quad \to \quad & -\left(4\frac{r_0}{\epsilon}\right)^{3/2} \dot{\epsilon}^2 + \left(4\frac{r_0}{\epsilon}\right)^{1/2} (\nabla\epsilon)^2 = \\ &= \frac{32\sqrt{r_0}}{9} \left(-4r_0 \frac{\dot{z}^2}{z^{4/3}} + (\nabla z)^2 \right) \end{aligned} \tag{5.6}$$

with $\epsilon = z^{4/3}$. A similar thing happens for the S^5 coordinates. For the gauge fields

$$L_F \sim \sqrt{\frac{r_0}{\epsilon}} \vec{E}^2 + \sqrt{\frac{\epsilon}{r_0}} \vec{B}^2 \tag{5.7}$$

and near the horizon $\vec{E} \to 0$. Since $\vec{E}$ is the source for fundamental strings attached to the brane, that implies that such couplings are suppressed close to the horizon. Taking the near horizon limit does not essentially modify the kinetic terms we have described above.

The effective kinetic terms can be examined at large u in the same way done for the potential. The corrections are power series in $(TR^2/u)^4$ and they start at two loops. An open string calculation similar to that for the potential can be done also for the kinetic terms along the lines of [34, 32]. The result for the one-loop correction to the two-derivative terms is

$$F_2 \sim -\frac{N}{8\cdot 16\pi^2}\int_0^\infty \frac{dt}{t} e^{-\frac{r^2 t}{2\pi\alpha'}} \vartheta_2\left(\frac{i\beta^2}{\alpha'\pi t}\right) f_{\mathrm{open-string}}(t)\; g_2(t) \tag{5.8}$$

with

$$g_2(t) = 3\frac{\vartheta_2''(it)}{\vartheta_2(it)} - 4\frac{\vartheta_1'''(it)}{\vartheta_1'(it)} = 2\pi^2 + \mathcal{O}(e^{-2\pi t})\,. \tag{5.9}$$

In the near-horizon limit, this becomes

$$\begin{aligned} F_2^M &= -\frac{N}{64}\int_0^\infty \frac{dt}{t} e^{-\frac{u^2\beta^2 t}{2\pi^2}} \vartheta_2\left(\frac{i}{t}\right) \\ &= -\frac{N}{64}(2\pi)^{3/4}(u\beta)^{-1/4} e^{-u\beta/\sqrt{2\pi}} + \cdots \end{aligned} \tag{5.10}$$

where we assumed in the second line $u\beta \gg 1$. Thus, at one-loop the conclusion is that the correction to the kinetic terms is exponentially suppressed for large u (as for the potential). This is again a signal of softly broken supersymmetry. Also, the insertion of the two vertex operators in the diagram reduces the effective supersymmetry to $\mathcal{N}=2$ so that already at two loops there is a power correction. It is also obvious from our expression that if the massive string is not thermal then the one-loop contribution to the kinetic terms vanishes.

If one continues to the F^4 couplings, then there is already a one-loop contribution in the extremal limit, and it is corrected by a power series in $(TR^2/u)^4$ coming from higher loops. The D-brane action (5.1) gives

$$S_{F^4} = -(2\pi\alpha')^4 T_3 \int \frac{H(r)}{8f(r)^{3/2}}\left[(F^2)^2 - F^4\right], \tag{5.11}$$

where

$$F^4 = F_{\mu\nu}F^{\mu}{}_{\rho}F^{\rho}{}_{\sigma}F^{\nu\sigma}\,, \qquad F^2 = \frac{1}{2}F_{\mu\nu}F^{\mu\nu} \tag{5.12}$$

and the contractions above are made with the effective metric in $\tilde{g}$. We have

$$F^2 = \vec{B}^2 - \frac{1}{f}\vec{E}^2\,, \qquad (F^2)^2 - \frac{1}{2}F^4 = -\frac{2}{f}(\vec{E}\cdot\vec{B})^2 \tag{5.13}$$

In the near-horizon limit, it becomes

$$S_{F^4} = 2\pi^2 N\int d^4x \frac{(\vec{E}\cdot\vec{B})^2}{u^4\left(1-\frac{u_0^4}{u^4}\right)^{5/2}} = 2\pi^2 N\int d^4x \frac{(\vec{E}\cdot\vec{B})^2}{u^4} + \mathcal{O}(T^4R^8)\,. \tag{5.14}$$

The leading term is the one-loop contribution while the next term appears at three loops.

The one-loop D-brane contribution to the F^4 coupling is

$$F_4 = \frac{N\alpha'^2}{2}\int_0^\infty dt\; t\; e^{-\frac{r^2 t}{2\pi\alpha'}}\left[\vartheta_3\left(\frac{i\beta^2}{\alpha'\pi t}\right) + \vartheta_2\left(\frac{i\beta^2}{\alpha'\pi t}\right) f_{\text{open}-\text{string}}(t)\; g_4(t)\right], \tag{5.15}$$

with

$$\begin{aligned} g_4(t) &= \frac{1}{\pi^4}\left[15\frac{\vartheta_2''''(it)}{\vartheta_2(it)} + 32\left(\frac{\vartheta_1'''(it)}{\vartheta_1'(it)}\right)^2 - 48\frac{\vartheta_1^{(5)}(it)}{\vartheta_1'(it)} - 120\frac{\vartheta_2''(it)}{\vartheta_2(it)}\frac{\vartheta_1'''(it)}{\vartheta_1'(it)}\right] \\ &= -121 + \mathcal{O}(e^{-2\pi t}) \end{aligned} \tag{5.16}$$

becoming in the near-horizon limit

$$F_4^M = \frac{N\beta^4}{2\pi^2}\int_0^\infty dt\, te^{-\frac{u^2\beta^2 t}{2\pi^2}}\left[\vartheta_3\left(\frac{i}{t}\right) - 8\cdot 121\vartheta_2\left(\frac{i}{t}\right)\right] = \frac{2\pi^2 N}{u^4} + \mathcal{O}\left(e^{-u\beta/\sqrt{2\pi}}\right), \tag{5.17}$$

which agrees with the leading order in the supergravity calculation. We can conclude that all effects that vanish in the limit of exact supersymmetry are exponentially suppressed in the presence of non-zero temperature.

We can use the supergravity calculation to study the effective electric and magnetic couplings of the U(1) obtained after the breaking of $\mathrm{SU}(N+1) \to \mathrm{SU}(N) \times \mathrm{U}(1)$.[4] In fact, since there is a potential for u, we cannot make this discussion when the consider the U(1) quantum theory. So, we consider integrating out the $\mathrm{SU}(N)$ degrees of freedom, and treat the U(1) sub-theory as a source theory that is renormalized. In the near-horizon limit[5] S_1^F implies the following electric and magnetic effective couplings

$$\frac{1}{g_e^2(T,u)} = \frac{1}{2\pi g_s f(u)}, \qquad \frac{1}{g_m^2(T,u)} = \frac{f(u)}{2\pi g_s}, \tag{5.18}$$

with $f(u) = \sqrt{1-(\pi T\lambda^{1/2}/u)^4}$, and $\lambda = 2Ng_{\mathrm{YM}}^2$ is the 't Hooft coupling. The equations above can be rewritten in terms of the associated 't Hooft couplings as

$$\lambda_e(T,u) = \lambda f(u)\,, \qquad \lambda_m(T,u) = \frac{\lambda}{f(u)}, \qquad \lambda_e(T,u)\lambda_m(T,u) = \lambda^2\,. \tag{5.19}$$

The electric field component E_i is the source for fundamental strings in the x^i direction. The magnetic field B_i is the source for D-strings in the x_i direction. S duality of the IIB theory, interchanges F and D strings and explains why $\lambda_m/\lambda = \lambda/\lambda_e$.

We can view here the Higgs expectation value as the running scale and the temperature as the cutoff (equivalently as the Λ scale).[6] The effective couplings

[4]As mentioned earlier our results are valid for the breaking $\mathrm{SU}(N+M) \to \mathrm{SU}(N)\times\mathrm{SU}(M)\times\mathrm{U}(1)$ with $M \ll N$.

[5]We assume imaginary time.

[6]From the gauge theory point of view, the correct scale is ρ in (3.8). However, using u does not change the qualitative behavior of the effective couplings.

satisfy the renormalization group equations

$$u\frac{\partial\lambda_e}{\partial u}=\beta_e(\lambda_e)=-2\lambda_e+2\lambda_m \quad , \quad u\frac{\partial\lambda_m}{\partial u}=\beta_m(\lambda_m)=2\lambda_m-2\frac{\lambda_m^2}{\lambda_e}\,, \tag{5.20}$$

with the ultraviolet coupling $\lambda_e(u=\infty)=\lambda_m(u=\infty)=\lambda$. As one flows to the infrared, the electric coupling decreases while the magnetic coupling decreases. When $u=\pi T\lambda^{1/2}$ (the horizon in supergravity) the electric coupling vanishes while the magnetic coupling blows up. Thus, the electric coupling is IR free while the magnetic coupling is IR strong. The region inside the horizon describes the theory for energies below the thermal cutoff. The region far from the horizon corresponds to scales were supersymmetry is a good approximation.

We can analyze the system of β-functions (5.20) beyond our specific solution. The general solution can be written in the form

$$\lambda_e^2=C_1+\frac{C_2}{u^4}\,, \qquad \lambda_m^2=\frac{C_1^2}{C_1+\frac{C_2}{u^4}} \tag{5.21}$$

Note that $C_1\geq 0$ if we require a reasonable UV limit. There are different types of behavior:

- $C_1=0$, then $C_2>0$ and $\lambda_m=0$ and λ_e is an asymptotically free coupling that blows up in the IR.
- $C_1>0$ and $C_2<0$. This behavior is the one realized in the D-brane system with $\lambda^2=C_1$ and $C_2=-\pi^4T^4\lambda^2$.
- $C_1,C_2=0$. This the generic fixed point of this system of β-functions with $\lambda_e=\lambda_m=$ constant.
- $C_1>0$, $C_2>0$. This situation is qualitatively similar to the $C_1=0$ case but the electric coupling has a non-zero UV value, while the magnetic coupling is IR free.

The running effective couplings in (5.19) are the physical effective U(1) couplings (as we argued in the previous section). They constitute concrete predictions of supergravity for spontaneously broken Yang-Mills theory in the large-N limit. It would be interesting if they could be compared to a different approach.

6. Phenomenological implications

There are two effects presented above that merit an extra discussion. The first is the fact that the leading behavior of the vacuum energy induced in the probe brane scales as T^8/u^4. To decipher this dependence we will have to remind the reader the behavior of the vacuum energy in a spontaneously broken supersymmetric theory.

The vacuum one-loop diagram (in four dimensions) has a well-known dependence on the cutoff as well as the particle mass

$$\int \frac{d^4k}{\pi^2}\log(k^2+m^2) = \frac{\Lambda^4}{2}\left(\log\Lambda^2-\frac{1}{2}\right)+m^2\Lambda^2+\frac{m^4}{2}\left(\log\frac{m^2}{\Lambda^2}-\frac{1}{2}\right)- \\ -\frac{1}{3}\frac{m^6}{\Lambda^2}+\frac{1}{8}\frac{m^8}{\Lambda^4}+\mathcal{O}(\Lambda^{-6}) \tag{6.1}$$

In a supersymmetric theory the leading Λ^4 terms are proportional to Str[1] = # fermions-# bosons. The subleading Λ^2 terms are proportional to Str$[m^2]$. In a spontaneously broken global theory the soft masses are proportional to the helicities so that mass supertraces turn into helicity supertraces [35, 36]. For an $\mathcal{N}=4$ theory Str$[m^2]=0$ and the leading term is proportional to $M^4_{\rm SUSY}$. For an $\mathcal{N}=8$ theory (supergravity) with a cutoff, softly broken so that masses are again proportional to helicities Str$[m^4]$ = Str$[m^6]=0$ but the logarithmic divergence prohibits a further suppression of the vacuum energy. For Scherk-Schwarz [27] supersymmetry breaking in string theory [28] as well as that induced by temperature (in string theory this is similar to the previous mechanism) the vaccum energy for spontaneously broken $\mathcal{N}\geq 4$ supersymmetry (in four dimensions) scales as $M^4_{\rm SUSY}$ [38].

There has been a revival lately of the the idea [37] that our four dimensional universe is a three-brane, embedded in ten dimensional, (partially) compactified space-time. This was viable in situations with a very low string scale [38]–[48]. Moreover, other three-branes may be providing mirror universes. A prototype of this is the Hořava-Witten interpretation of the Heterotic String [26]. A popular mechanism of supersymmetry breaking in our universe is that supersymmetry breaks spontaneously (probably due to strong dynamics) on a spectator brane and then this supersymmetry breaking is communicated via gravity to the universe brane [49].

Here we have a toy model of this situation. The spectator brane is the black-brane while our universe is represented by the probe brane. Supersymmetry is broken in the far-away brane by thermal effects. Although this might not be the exact way we would like supersymmetry to be broken, it does represents soft supersymmetry breaking. Both three-branes carry $\mathcal{N}=4$ supersymmetry. There is only one dynamical scale in the problem (when supersymmetry is unbroken): the distance or Higgs expectation value, $M_{\rm DYN}\sim u$. The supersymmetry breaking scale is given by the temperature, $M_{\rm SUSY}\sim T$.

When $M_{\rm SUSY}\sim M_{\rm DYN}$ then supersymmetry is strongly broken and we have obtained for the vacuum energy the result (3.9) which scales as $M^4_{\rm SUSY}$. This is the natural expectation, as argued above, from broken $\mathcal{N}=4$ supersymmetry. In the opposite limit where the supersymmetry breaking scale is much smaller than the dynamical scale, we find that the cosmological constant induced in our universe due to supersymmetry breaking on the spectator brane is much smaller: it scales as $\sim M^8_{\rm SUSY}/M^4_{\rm DYN}$. This scaling is subleading to that expected in a finite theory with maximal supersymmetry as argued previously.

One comment is in order here. We consider the probe brane to be initially at zero temperature (unbroken supersymmetry). The statement that the vaccum nergy induced on the brane is supressed is equivalent to the statement that the probe brane does not thermalize at any given finite time. In fact we do expect this to be true, since we are in the limit of large distance. Since the interactions of the probe brane with the thermal pile of branes are of gravitational stregth we do not expect thermalization at finite times. Note that this intuition implies that quantum corrections on the probe brane will also be supressed.

In order to describe a concrete situation we assume that our universe is a Dp-brane a distance r away from the black Dp-brane. We assume that the $(9-p)$ transverse dimensions are compactified with volume V_t while the extra $p-3$ directions on the brane are compactified with volume $V_{||}$. Thus, the four-dimensional Planck scale and low energy gauge coupling are given by

$$M_{\rm P}^2 = \frac{V_t V_{||} M_{\rm s}^8}{g_{\rm s}^2}\,, \qquad \frac{1}{g_{\rm YM}^2} = \frac{V_{||} M_{\rm s}^{p-3}}{g_{\rm s}} \tag{6.2}$$

For the solutions of the previous section to be applicable here we must have that the distance between the branes is much smaller than the linear dimensions of the compact tranverse space:

$$\frac{r}{l_s} \ll \frac{V_t^{1/(9-p)}}{l_s} = \left[\left(\frac{M_{\rm P}}{M_{\rm s}}\right)^2 g_{\rm s} g_{\rm YM}^2\right]^{1/(9-p)} . \tag{6.3}$$

The effective induced cosmological constant on the universe-brane is given by

$$\frac{\Lambda_4}{M_{\rm P}^4} = \frac{V^{int}}{M_{\rm P}^4}\,, \tag{6.4}$$

where V^{int} is given in (2.15). In the cases we will be interested we can approximate[7]

$$\xi_4 \equiv \frac{\Lambda_4}{M_{\rm P}^4} \sim \frac{V_{||} M_{\rm s}^{p+1}}{g_{\rm s} M_{\rm P}^4} \left(\frac{r_0^2}{rL}\right)^{7-p} . \tag{6.5}$$

For $p<5$ we find

$$\xi_4 \sim \frac{V_{||} M_{\rm s}^{p+1}}{g_{\rm s} M_{\rm P}^4} \lambda^{\frac{2(9-p)}{(5-p)}} \frac{(T\ l_{\rm s})^{\frac{4(7-p)}{(5-p)}}}{(r\ M_{\rm s})^{7-p}} \sim \frac{\lambda^{\frac{2(9-p)}{(5-p)}}}{g_{\rm YM}^2} \left(\frac{M_{\rm s}}{M_{\rm P}}\right)^4 \frac{(T\ l_{\rm s})^{\frac{4(7-p)}{(5-p)}}}{(r\ M_{\rm s})^{7-p}} . \tag{6.6}$$

The supersymmetry breaking scale $M_{susy}$is identified with the temperature T. We will take it to be $T \sim \mathcal{O}(10^3)$ GeV. Taking into account (6.3) we can obtain the

[7]We neglet factors of two and $\pi' s$.

following bound

$$\xi_4 \gg \xi_0 = \frac{\lambda^{\frac{2(9-p)}{(5-p)}}}{g_{\rm s}^{\frac{(7-p)}{(9-p)}} g_{\rm YM}^{4\frac{(8-p)}{(9-p)}}} \left(\frac{M_{susy}}{M_{\rm s}}\right)^{\frac{4(7-p)}{(5-p)}} \left(\frac{M_{\rm s}}{M_{\rm P}}\right)^{4+2\frac{(7-p)}{(9-p)}} . \tag{6.7}$$

This gives a lower (order of magnitude bound) on the induced cosmological constant, namely ξ_0. One can make the cosmological constant two or more orders of magnitude bigger than the bound. For the cases of interest we have

$$\begin{aligned} \xi_0(p=3) &= \frac{\lambda^6}{g_{\rm s}^{1/2} g_{\rm YM}^{10/3}} \left(\frac{M_{susy}}{M_{\rm P}}\right)^8 \left(\frac{M_{\rm s}}{M_{\rm P}}\right)^{-8/3} , \\ \xi_0(p=4) &= \frac{\lambda^{10}}{g_{\rm s}^{2/3} g_{\rm YM}^{3}} \left(\frac{M_{susy}}{M_{\rm P}}\right)^{12} \left(\frac{M_{\rm s}}{M_{\rm P}}\right)^{-34/5} . \end{aligned} \tag{6.8}$$

Moreover ξ_0 becomes smaller when the string scale is of the same order as the Planck scale. Moreover $g_{\rm YM} \sim \mathcal{O}(1)$ and we will drop it from the formula. Putting in numbers we obtain

$$\xi_0(p=3) = 10^{-128} \ \frac{\lambda^6}{g_{\rm s}^{1/2}} , \qquad \xi_0(p=4) = 10^{-192} \frac{\lambda^{10}}{g_{\rm s}^{2/3}} \tag{6.9}$$

The cosmological constant can be made smaller by making the string couplings $g_{\rm s} \ll 1$, larger by making $N \gg 1$, or by making $r/l_{\rm s}$ arbitrarily small. In particular it should be noted that the cosmological constant can obtain easily a value close to 10^{-120}, the favorite number today. Moreover this can be accomodated even for $M_{\rm s} \sim M_{susy}$ at the expense of small values of $g_{\rm s}$.

As argued above it is expected that the extra contributions to the vacuum energy on the universe brane due to loops of brane fields will have extra suppression factors of the coupling constant. Moreover, there is an indication [30] that even $\mathcal{N} = 2$ supersymmetry on the branes might be enough to control in a similar fashion the vacuum energy in this context. We can investigate the situation where the stack of N branes are located at an orbifold sigularity and have reduced supersymmetry [50]. For a smooth such configurations the metric and five-form field have been obtained in [51]. The five-form field stregth is not modified whereas the only modification to the metric is in the geometry of the space that replaces S^5. Their black analogs can be easily written as in (2.1) with $p = 3$ where the metric of the five-sphere is replaced by the metric of a U(1) bundle on a Kähler-Einstein space [51]. The field stregth of the U(1) connection is equal to the Kähler two-form. Going through the same procedure as before we find the same potential as in the $N = 4$ case. This strongly suggests that the supression of the cosmological constant is due to $N = 1$ rather than $N = 4$ supersymmetry. The observations above may be important for controlling the scale of the cosmological constant of our universe after supersymmetry breaking.

The second issue to be commented upon concerns the fact that the effective velocity of light on the probe brane is field-dependent. This is due here to the black nature of the spectator branes.

A variable velocity of light has a similar effect for the evolution of the universe as that due to inflation. This possibility has been investigated recently [29] and might provide a viable alternative to inflation. One of the main problems in such an approach is to find a natural dynamical evolution of the velocity of light, rather than an ad hoc variability and to have a certain predictivity on the nature of interactions.

Our example (although rather a toy example) provides a concrete dynamical framework for a variable speed of light. The proposal here is that our brane-universe is the probe brane falling towards the black brane. Due to the fact that the effective velocity of light is distance dependent as in (5.5), it becomes smaller with time passing. Moreover, it induces a Robertson-Walker type of metric on the probe brane. This setup deserves further study in order to investigate the possibility of a concrete and realistic alternative to inflation.

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A. Calculation of the Dp-brane RR background field strength

In the string frame the RR field strength satisfies

$$\nabla^{\mu} F_{\mu\nu_1\cdots\nu_{p+1}} = 0\,. \tag{A.1}$$

This is also implied for $p = 3$ by the self-duality condition (2.9).

We parametrize the metric as

$$\begin{aligned} ds^2 &= ds^2_{AdS} + ds^2_S \\ ds^2_{AdS} &= g_{00}(r)dt^2 + g_{rr}(r)dr^2 + \sum_{i=1}^{p} g_i(r)dx_i^2 \\ ds^2_S &= g_S(r)h_{\alpha\beta}dy^{\alpha}dy^{\beta}\,, \end{aligned} \tag{A.2}$$

where $h_{\alpha\beta}$ is the metric of the unit S^{8-p} sphere. The only non-zero components are $F_{r012\cdots p}$, except for $p = 3$ where the self-duality condition implies also that F_{45678} (in the S^5 directions) is non-zero.

Specified to the background of the form (A.2), eq. (A.1) gives

$$\left[\partial_r - \partial_r \log\left(\frac{V_{AdS}}{V_S}\right)\right] F_{r012\cdots p} = 0\,, \tag{A.3}$$

where $V_{AdS} = \sqrt{g_{00}g_{rr}\prod_{i=1}^{p} g_i}$ and $V_S = [g_S(r)]^{(8-p)/2}$. The solution is

$$F_{r012\cdots p} = c\frac{V_{AdS}}{V_S}\,, \tag{A.4}$$

where c does not depend on r. The dual field strength can be obtained using (2.9),[8]

$$F^{(p+1)\cdots 8} = \frac{c}{V_S^2\sqrt{h}}\,, \qquad F_{(p+1)\cdots 8} = c\sqrt{h}\,. \tag{A.5}$$

The constant c is related to the charge N by the Gauss' law,

$$\int_{S^{8-p}} F_{(p+1)\cdots 8} = -2\kappa_{10}^2 T_p N\,. \tag{A.6}$$

The coupled p-form–gravity field equations impose the relation eq.(2.4) between the charge N and the parameters L and r_0, so that

$$c = -\frac{2\kappa_{10}^2 T_p N}{\Omega_{8-p}} = (p-7)L^{(7-p)/2}\sqrt{r_0^{7-p} + L^{7-p}}\,. \tag{A.7}$$

For the black Dp-brane metric (2.1) we obtain

$$F_{r012\cdots p} = (p-7)\frac{L^{(7-p)/2}\sqrt{r_0^{7-p} + L^{7-p}}}{H_p(r)^2} \tag{A.8}$$

and integrating once and adjusting the constant of integration so that the p-form falls off at infinity we obtain

$$C_{012\cdots p} = \sqrt{1 + \frac{r_0^{7-p}}{L^{7-p}}}\frac{H_p(r) - 1}{H_p(r)}\,, \tag{A.9}$$

as advertised.

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